7-3: Similar Triangles

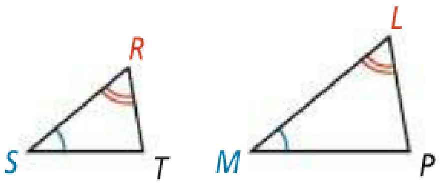
Learning objectives:

- Use the AA (angle-angle) postulate and the SAS and SSS theorems
- Use similarity to find indirect measurements

Jan 27-7:22 PM

The AA (angle-angle) Postulate

If . . .	Then . . .
$\angle S \cong \angle M$ and $\angle R \cong \angle L$	$\triangle SRT \sim \triangle MLP$



Q: why do you think this is true?
(Hint: think about trig ratios!)

Jan 27-7:25 PM

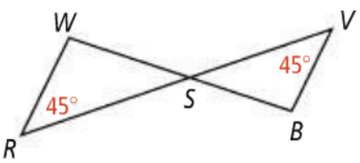
A: this is true because ***if the angles are congruent then their ratios for sine, cosine and tangent must be the same***.

Those ratios are simply one side over another (opposite over adjacent, for example), which is the definition of similar polygons (rule #2) that all side ratios must be equal.

For example, the sine of 30 degrees is 1/2. That means the side ratio must be 1/2 for ALL 30 degree angles.

Jan 27-7:27 PM

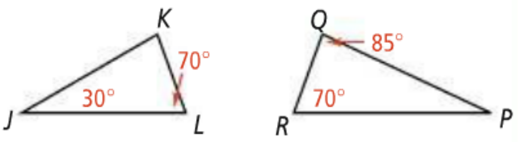
Are the two triangles similar? How do you know?



A **$\triangle RSW$ and $\triangle VSB$**

$\angle R \cong \angle V$ because both angles measure 45°.
$\angle RSW \cong \angle VSB$ because vertical angles are congruent.
So, $\triangle RSW \sim \triangle VSB$ by the AA $\sim$ Postulate.

B **$\triangle JKL$ and $\triangle PQR$**



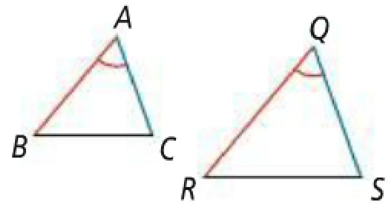
$\angle L \cong \angle R$ because both angles measure 70°.
By the Triangle Angle-Sum Theorem,
$m\angle K = 180 - 30 - 70 = 80$ and
$m\angle P = 180 - 85 - 70 = 25$. Only one pair of angles is congruent. So, $\triangle JKL$ and $\triangle PQR$ are *not* similar.

Jan 27-7:32 PM

Theorem 7-1

If . . .

$\frac{AB}{QR} = \frac{AC}{QS}$ and $\angle A \cong \angle Q$



Then . . .

$\triangle ABC \sim \triangle QRS$

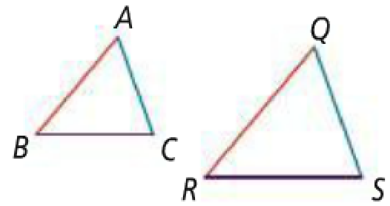
This is SAS for similarity! The difference is the "S" means *side ratio*.

Jan 27-7:33 PM

Theorem 7-2

If . . .

$\frac{AB}{QR} = \frac{AC}{QS} = \frac{BC}{RS}$



Then . . .

$\triangle ABC \sim \triangle QRS$

This is SSS for similarity! Again, the "S" means *side ratio*.

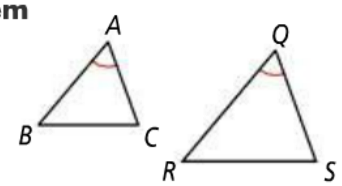
Jan 27-7:33 PM

Proof of theorem 7-1

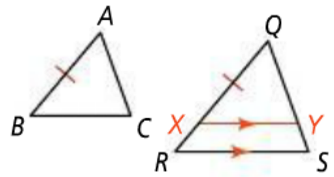
Proof of Theorem 7-1: Side-Angle-Side Similarity Theorem

Given: $\frac{AB}{QR} = \frac{AC}{QS}$, $\angle A \cong \angle Q$

Prove: $\triangle ABC \sim \triangle QRS$



Plan for Proof: Choose X on $\overline{RQ}$ so that $QX = AB$. Draw $\overleftrightarrow{XY} \parallel \overline{RS}$. Show that $\triangle QXY \sim \triangle QRS$ by the AA ~ Postulate. Then use the proportion $\frac{QX}{QR} = \frac{QY}{QS}$ and the given proportion $\frac{AB}{QR} = \frac{AC}{QS}$ to show that $AC = QY$. Then prove that $\triangle ABC \cong \triangle QXY$. Finally, prove that $\triangle ABC \sim \triangle QRS$ by the AA ~ Postulate.

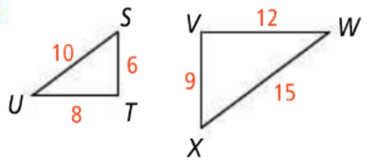


Try to work this out on your own! It will be on the quiz!

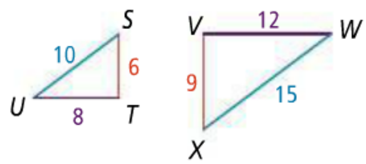
Jan 27-7:37 PM

Are the triangles similar? If so, write a similarity statement for the triangles.

A



Use the side lengths to identify corresponding sides.
Then set up ratios for each pair of corresponding sides.

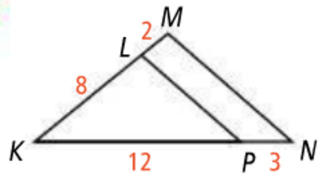


Shortest sides	$\frac{ST}{XV} = \frac{6}{9} = \frac{2}{3}$
Longest sides	$\frac{US}{WX} = \frac{10}{15} = \frac{2}{3}$
Remaining sides	$\frac{TU}{VW} = \frac{8}{12} = \frac{2}{3}$

All three ratios are equal, so corresponding sides are proportional.
$\triangle STU \sim \triangle XVW$ by the SSS ~ Theorem.

Jan 27-7:41 PM

B

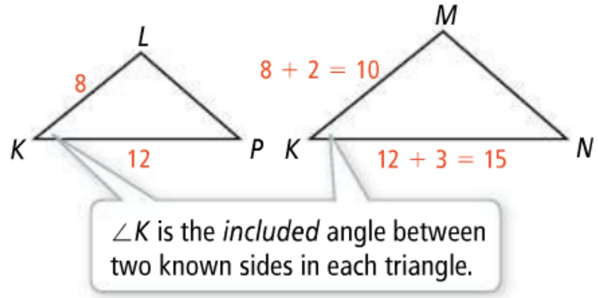


It can help to draw overlapping triangles separately

$\angle K \cong \angle K$ by the Reflexive Property of Congruence.

$\frac{KL}{KM} = \frac{8}{10} = \frac{4}{5}$ and $\frac{KP}{KN} = \frac{12}{15} = \frac{4}{5}$.

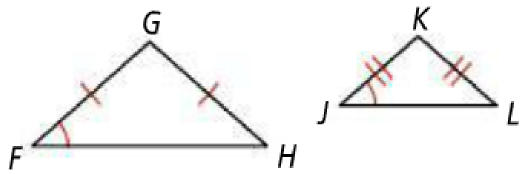
So, $\triangle KLP \sim \triangle KMN$ by the SAS $\sim$ Theorem.



Jan 27-7:45 PM

Given: $\overline{FG} \cong \overline{GH}$, $\overline{JK} \cong \overline{KL}$, $\angle F \cong \angle J$

Prove: $\triangle FGH \sim \triangle JKL$



Statements	Reasons
1) $\overline{FG} \cong \overline{GH}$, $\overline{JK} \cong \overline{KL}$	1) Given
2) $\triangle FGH$ is isosceles. $\triangle JKL$ is isosceles.	2) Def. of an isosceles $\triangle$
3) $\angle F \cong \angle H$, $\angle J \cong \angle L$	3) Base $\angle$s of an isosceles $\triangle$ are $\cong$.
4) $\angle F \cong \angle J$	4) Given
5) $\angle H \cong \angle J$	5) Transitive Property of $\cong$
6) $\angle H \cong \angle L$	6) Transitive Property of $\cong$
7) $\triangle FGH \sim \triangle JKL$	7) AA $\sim$ Postulate

Jan 27-7:48 PM

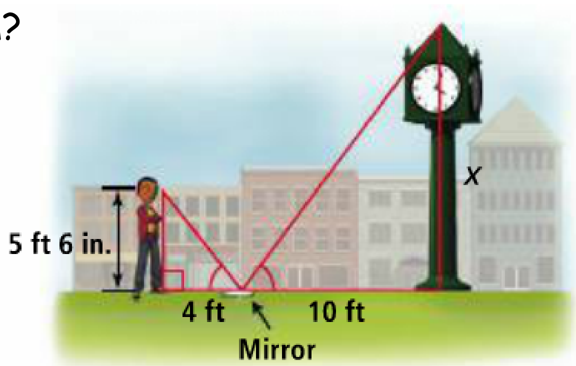
Essential Understanding Sometimes you can use similar triangles to find lengths that cannot be measured easily using a ruler or other measuring device.

You can use **indirect measurement** to find lengths that are difficult to measure directly. One method of indirect measurement uses the fact that light reflects off a mirror at the same angle at which it hits the mirror.

Note: we've done this before with ratios and trig functions!

Jan 27-7:51 PM

How would you solve this problem?



Jan 27-7:54 PM